APS Homework 1: Divide-and-Conquer

Problem 1: A Fake among 33 Coins

There are \( n = 33 \) identical-looking coins. 32 of the coins are genuine and all weigh the same, and 1 coin is fake and weighs slightly less than the genuine coins. Given only a two-pan balance scale (Fig. 1) and the 33 coins, which coin is fake?

![Figure 1. A two-pan balance scale.](image)

**Problem 1a:** What is the minimum number of weighings needed to determine with 100% certainty which of the 33 coins is fake in the worst-case scenario?

**Problem 1b:** Describe a Divide-and-Conquer algorithm for determining with 100% certainty which of the 33 coins is fake in the minimum number of weighings.

**Problem 1c:** Generalize the algorithm you provided in Problem 1b to work for any arbitrary number of coins \( n > 0 \).

**Problem 1d:** Prove that the algorithm you provided in Problem 1c is correct for any \( n > 0 \).

**Problem 1e:** As a function of \( n \), what is the minimum number of weighings needed to determine with 100% certainty which of the \( n \) coins is fake in the worst-case scenario?

Problem 2: Binary Search

You are given a list `ints` containing \( n = |\text{ints}| = 8 \) integers in ascending order (i.e., 8 integers ordered from smallest to largest). Given an arbitrary integer \( x \), does `ints` contain \( x \)? Define a “comparison” to be a procedure that, given 2 integers \( a \) and \( b \), tells you if \( a > b \), \( a < b \), or \( a = b \).
**Problem 2a:** What is the minimum number of comparisons needed to determine with 100% certainty if \( \text{ints} \) contains \( x \) in the worst-case scenario?

**Problem 2b:** Describe a Divide-and-Conquer algorithm for determining with 100% certainty if \( \text{ints} \) contains \( x \).

**Problem 2c:** Generalize the algorithm you provided in Problem 2b to work for any sorted list of any arbitrary size \( n > 0 \).

**Problem 2d:** Prove that the algorithm you provided in Problem 2c is correct for any \( n > 0 \).

**Problem 2e:** As a function of \( n \), what is the minimum number of comparisons needed to determine with 100% certainty if an arbitrary sorted list of integers contains \( x \) in the worst-case scenario?